RESEARCHERID
THOMSON REUTERS

[Sumalatha * *et al.*, 7(1): January, 2018]
IC™ Value: 3.00

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

POISEUILLE FLOW OF A JEFFREY FLUID IN AN INCLINED ELASTIC TUBE

B. Sumalatha[*1] & S. Sreenadh[2]
[*1&2]Department of Mathematics, Sri Venkateswara University, Tirupati

DOI: 10.5281/zenodo.1135888

ABSTRACT
The flow of a Jeffrey fluid in an inclined elastic tube is investigated. The expressions for velocity and flux flow rate are determined. The flux is determined as a function of inlet, outlet, external pressures, non-Newtonian Jeffrey parameter and the elastic parameters of the tube. The effects of different parameters on the velocity and the flux are discussed. It is observed that the effects of Jeffrey parameter and elastic parameters have strong effects on the pumping phenomena. When the Jeffrey parameter and inclination parameter tends to zero, our results agree with those of Rubinow and Keller [J. Theor. Biol. 35, 299 (1972)]. Further the increasing elastic parameters and inclination parameter increase the flux of the non-Newtonian fluid in the elastic tube

KEYWORDS: Jeffrey fluid, elastic tube, Inlet pressure, outlet pressure and inclination parameter.

I. INTRODUCTION

There has been a recent interest in the study of biofluid flows through elastic tubes and channels because of their important biological and industrial applications. In particular the study of blood flows through an artery plays an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular diseases. Hence, over the past years many laboratory experiments have been conducted to investigate the properties of fluid flow through tubes. Among them Young [1] was the first who pointed out the importance of elasticity for pulse wave originating from the heart. Later, the results of Rubinow and Keller [2] exhibit the principle non-linear features of the pressure-flow relations observed in veins. The first experimental studies on internal flows through conduits bounded by soft walls were carried out by Lahav et al. [3] and Krindel & Silberberg [4]. They conducted experiments in gel-walled tubes of sub-millimeter diameter and found that the flow rate through the tube is smaller than the flow rate expected for a rigid tube for the same pressure difference. Recently experimental studies on the flow through soft tubes and channels were done by Kumaran [5]. Mark [6] gave simultaneous solution for the Navier-Stokes and elastic membrane equations by a finite element method. Womersley [7] studied oscillatory flows in Arteries and gave a constrained elastic tube as a model of arterial flow and pulse transmission. Erbay et al. [8] explained finite axisymmetric deformations of elastic tubes with an approximate method. Dalin Tang et al. [9] gave a mathematical model with free moving boundary to study viscous flow in axisymmetric collapsible tubes with three kinds of stenosis subjected to prescribed pressure drop and uniform external pressure conditions.

Under several investigations in the area of fluid dynamics, the Poiseuille law is consider to be very important as it explains the linear relation between the flux and the pressure difference between the ends of the tube in the case of viscous incompressible fluid in a rigid tube. However, in the vascular beds of mammals, the pressure flow relation is always nonlinear. This non-linearity is due to the elastic nature of the blood vessels. Hence, to analyze the effect of elasticity on small blood vessels, a mathematical model was designed by Vajravelu et al. [10] to study the flow of Herschel-Bulkley fluid in an elastic tube. Vajravelu et al. [11] made an attempt to study the peristaltic transport of a Herschel-Bulkley fluid in an inclined tube and Taha Sochi [12] developed a model for the Newtonian and Power-law fluids using two pressure-area elastic relations to describe the tube dispensability. Sreenadh et al. [13] analyzed the flow of a Casson fluid through an inclined tube of non-uniform cross section with multiple steosis. Ravi Kumar et al. [14] studied the unsteady peristaltic pumping in a finite length tube with permeable wall.

A non-Newtonian fluid model that has attracted many researchers is the Jeffrey fluid as this is found to be a better model for physiological fluids (Vajravelu et al. [17], Hayat et al. [15]). Jeffrey fluid model is a significant

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

generalization of Newtonian fluid model as the later one can be deduced as a special case of the former. Several researchers have studied Jeffrey fluid under different conditions. Sreenadh et al. [16] studied the unsteady flow of a Jeffrey fluid in an elastic tube with stenosis. Vajravelu et al. [17] investigated the influence of heat transfer on peristaltic transport of a Jeffrey fluid. Jyothi et al. [18] have considered the pulsatile flow of a Jeffrey fluid in a circular tube lined internally with porous material. Kothandapani and Srinivas [19] and Pandey et al. [20] have considered the peristaltic motion of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. Kavitha et al. [21] investigated peristaltic transport of a Jeffery fluid in contact with a Newtonian fluid in an inclined channel. Recently, Sreenadh et al. [22] studied the Effect of elasticity on Hagen-Poiseuille flow of a Jeffrey fluid in a tube.

The main objective of the present work is to analyze the effect of elasticity on a non-Newtonian fluid Jeffrey fluid flow in an inclined elastic tube. The expressions for velocity and flux flow rate are determined. The variations of flux with elastic parameters are discussed.

II. BASIC EQUATIONS

The constitutive equations for an incompressible Jeffrey fluid are

$$\bar{T} = p\bar{I} + \bar{S}$$

$$\bar{S} = \frac{\mu}{1+\lambda_1}(\dot{\gamma} + \lambda_2 \ddot{\gamma}) \qquad (1)$$

where $\bar{T}$ is the cauchy's stress tensor, $\bar{S}$ is the extra tensor , $\bar{I}$ is the identity tensor, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time γ is the shear rate and dots over the quantities indicate differentiation with respect to time.

III. FORMUATION OF THE PROBLEM

Consider the Poiseuille flow of a steady laminar incompressible Jeffrey fluid in an elastic tube inclined at an angle α with the horizontal, the radius of the tube varies due to the elastic property of the tube and is assumed to be $a(z)$. Let L denotes the length of the tube. The blood is modeled as a non-Newtonian Jeffrey fluid and the flow is axisymmetric. The axisymmetric geometry facilitates the choice of the cylindrical polar coordinate system (r, ϕ, z), where r and z denotes the radial and axial coordinates and ϕ is the azumuthal angle. The momentum equation governing the flow is

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) = -\mu\frac{\partial p}{\partial z} + \rho g \sin\alpha \qquad (2)$$

where τ_{rz} is the shear stress of the Jeffrey fluid and is given by

$$\tau_{rz} = \frac{\mu}{1+\lambda_1}\left(-\frac{\partial u}{\partial r}\right) \qquad (3)$$

here u is the axial velocity, p is the pressure and α is the inclination parameter.
The boundary conditions are

τ_{rz} is finite at $r = 0$ (4)

$u = 0$ at $r = a(z)$ (5)

IV. SOLUTION OF THE PROBLEM

To solve the equations (2) and (3) under the boundary conditions (4) and (5), we make use of the following non-dimensional quantities

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

$$\bar{r} = \frac{r}{a_0},\ \bar{a} = \frac{a}{a_0},\ \bar{z} = \frac{z}{L},\ \bar{u} = \frac{u}{U}$$

$$\bar{p} = \frac{a_0}{L\mu U}p,\ \bar{\tau}_{rz} = \frac{\tau_{rz}}{\mu\left(U/a_0\right)} \tag{6}$$

where $P = -\frac{\partial p}{\partial z}$, a_0 is the radius of the tube in the absence of elasticity, L is the length of the tube, U is the average velocity. The governing equation (after dropping the bars) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right) = -\frac{\partial p}{\partial z} + \frac{\sin\alpha}{F} \tag{7}$$

where $F = \frac{\mu U}{a_0^{\ 2}\rho g}$

The non-dimensional boundary conditions are as follows:

τ_{rz} is finite at $r = 0$ (8)

$u = 0$ at $r = a(z)$ (9)

Solving equations (7) subject to the boundary conditions in (8) and (9), we obtain the velocity field as

$$u = \frac{(1+\lambda_1)(P+f)}{4}[a^2 - r^2]\ , \tag{10}$$

where $$f = \frac{\sin\alpha}{F}$$

The volume flux Q through any cross section is given by

$$Q = 2\int_0^a urdr$$

$$= \frac{P+f}{8}[1+\lambda_1]a^4 \tag{11}$$

The above equation (11) gives the volume flux for an inclined tube of varying radius 'a'. Using the fact that the variation occurs due to elasticity of the tube wall, we assume the Poiseuille law for Jeffrey fluid flow in an inclined elastic tube and discuss the consequences in the next section.

V. THEORETICAL DETERMINATION OF FLUX

We now calculate theoretically the study flux Q of an incompressible Jeffrey fluid of viscosity μ in an inclined elastic tube of radius a(z) and length L. We assume that the fluid enters the tube with the pressure p_1 and leaves it with lower pressure p_2 , while the pressure outside the tube is p_0 if z denotes the distance along the tube from inlet end, then the pressure p(z) in the fluid at z decreases from p(0)= p_1 to p(1)= p_2 as a consequence of the pressure difference p(z)- p_0 between the inside and outside of the tube, the tube may expand or contract, and hence the shape of its cross section may deform due to the elastic property of the wall. Therefore the conductivity σ_1 of the tube at z will depend on the pressure difference. We consider $\sigma_1 = \sigma_1$ (p(z) - p_0) as a known function of p(z) - p_0 .This conductivity is assumed to be same as that of a uniform tube having the same cross section as that of at z. We assume that the Q is related to the pressure gradient by the relation

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

$$Q = \sigma_1 (p - p_0)(P + f) \tag{12}$$

where $$\sigma_1 (p - p_0) = \frac{1}{8}(1 + \lambda_1) a^4 \tag{13}$$

Integrating equation (12) with respect to z from z=0 and using the inlet condition $p(0) = p_1$, we obtain

$$Qz = \int_{p(z)-p_0}^{p_1-p_0} \sigma_1 (p' \ ) dp' \ + \int_0^z \frac{1+\lambda_1}{8} a^4 dz \tag{14}$$

where $p' = p(z) - p_0$. This equation determines p (z) implicitly in terms of Q and z. To find Q, we set z=1and $p(1) = p_2$ in equation (14) to obtain

$$Q = \int_{p(1)-p_0}^{p_1-p_0} \sigma_1 (p' \ ) dp' \ + \int_0^1 \frac{(1+\lambda_1)}{8} a^4 dz \tag{15}$$

In the present case (circular cross section), the radius a is a function of $p - p_0$, that is, $a = a(p - p_0)$

Equation (15) can be written as

$$Q = \frac{1+\lambda_1}{8} \left[\int_{p_2-p_0}^{p_1-p_0} a^4 dp' + f a^4 \right] \tag{16}$$

If the hoop stress or tension T(a) in the tube wall is known as a function of a, then $a(p - p_0)$ is determined by the equilibrium condition

$$\frac{T(a)}{a} = p - p_0 \tag{17}$$

Application to flow through an artery is determined by the static pressure –volume relation of 4cm long piece of the human external iliac artery, and converted into a tension verses length curve. Using the least squares method, Rubinow and Keller [2] gave the following equation:

$$T(a) = t_1 (a-1) + t_2 (a-1)^5 \tag{18}$$

where $t_1 = 13$ and $t_2 = 300$

By substituting (18) in (17), on simplification we obtain

$$dp' = [\frac{t_1}{a_2} + t_2 (4a^3 - 15a^2 + 20a - 10 + \frac{1}{a^2})] da \tag{19}$$

using (19), (16) can be written as

$$Q = \frac{1}{8}(1+\lambda_1) \left[\int_{p_2-p_0}^{p_1-p_0} a^4 [\frac{t_1}{a_2} + t_2 (4a^3 - 15a^2 + 20a - 10 + \frac{1}{a^2})] da \ + \ f a (p_2 - p_0)^4 \right] \tag{20}$$

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

On further simplification, we get

$$Q = \frac{1}{8}(1+\lambda_1)\left[(g(a_1) - g(a_2)) + f{a_2}^4\right]$$

$$g(a) = t_1\frac{a^3}{3} + t_2(4\frac{a^8}{8} - 15\frac{a^7}{7} + 20\frac{a^6}{6} - 10\frac{a^5}{5} + \frac{a^3}{3})$$

where

$$a_1 = a(\mathrm{p}_1 - p_0) \qquad (21)$$
$$a_2 = a(\mathrm{p}_2 - p_0)$$

VI. RESULTS AND DISCUSSIONS

From equation (20), we have calculated the flux Q as a function of radius of the tube $a(p - p_0)$ for different values of elastic parameters t_1 and t_2, inclination of the tube α and Jeffrey parameter λ_1 for fixed pressure difference at outlet end of the tube $p_2 - p_0 = 10$. The results are shown in the Figures 1 to 4. From Figure 1and 2, it is observed that the flux increases with increasing elastic parameters t_1 and t_2. In particular the elastic parameter t_2 shows more effect on the flux when compared to t_1. In Figure 3, for a Jeffrey fluid in an inclined tube, we have noticed that for a given radius $a(p - p_0)$, the flux Q increases with increasing α for $\frac{\pi}{4} \le \alpha \le \frac{\pi}{2}$. From Figure 4, it is observed that the flux increases with the increment in the Jeffrey parameter λ_1 . In order to understand the effect of pressure difference at ends of the tube on the flux, we considered the variation of Q verses $p_1 - p_0$ for different values of $p_2 - p_0$ and variation of Q verses $p_2 - p_0$ for different values of $p_1 - p_0$ in Figures 5 and 6. From Figure 5 it is found that Q increases with the increasing values of $p_1 - p_0$. This phenomena exists due to the constant pressure difference $p_2 - p_0$. The opposite behaviour is observed in the case of $p_2 - p_0$ which is shown in Figure 6. From equation (10), we have calculated the velocity u as a function of radius of the tube r for different values of Jeffrey parameter λ_1 and inclination parameter α of the tube for fixed pressure gradient P=8 and pressure difference at inlet end of the tube. The results are shown in the Figures 7 to 10. Figures 7 and 8 depict that the velocity increases with the increasing values of Jeffrey parameter. The similar behaviour is observed regarding Jeffrey parameter by Vajravelu et al. [24] In particular, the flux takes maximum value in figure 8 compared to the figure 7. This is due to the increment in the inlet pressure and hence more pressure difference at the inlet end of the tube. From Figures 9 and 10, it is clear that for a given α , the velocity increases with the increment in r.

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

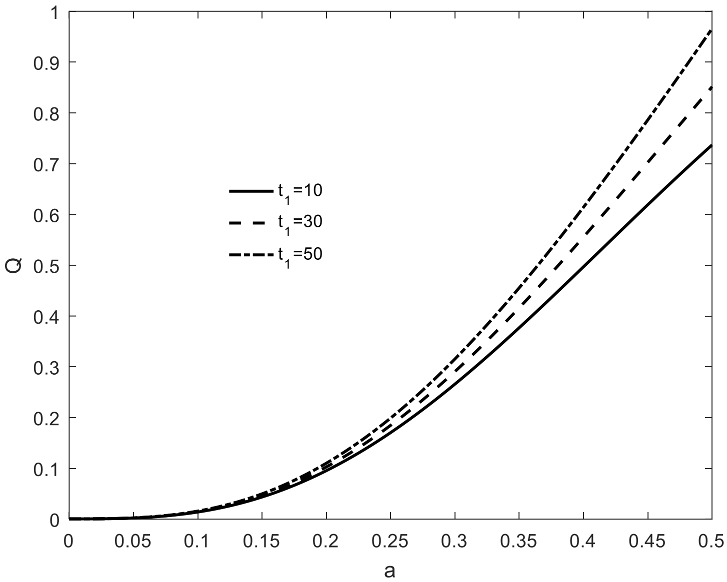

Figure 1. Variation of flux Q with radius a for different values of the elastic parameter t_1

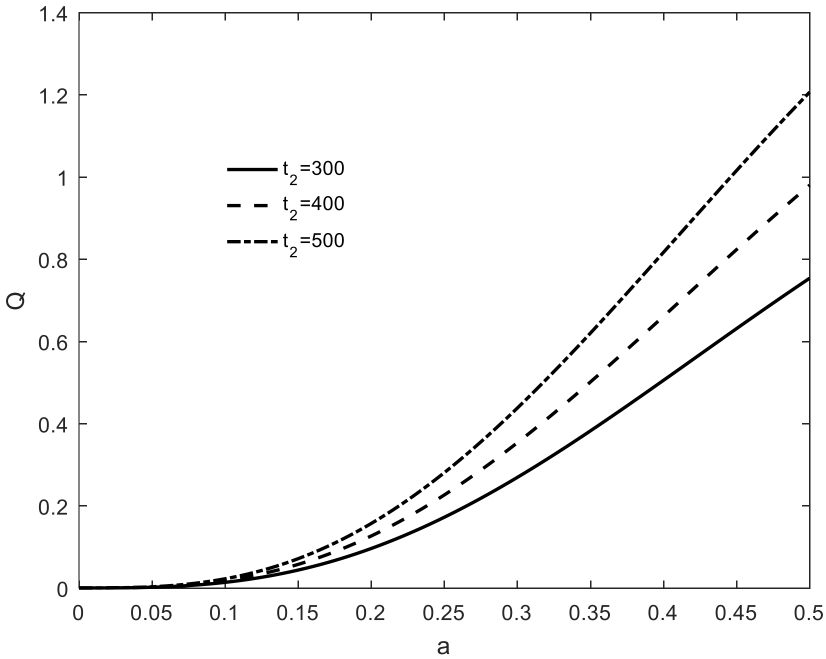

Figure 2. Variation of flux Q with radius a for different values of the elastic parameter t_2.

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

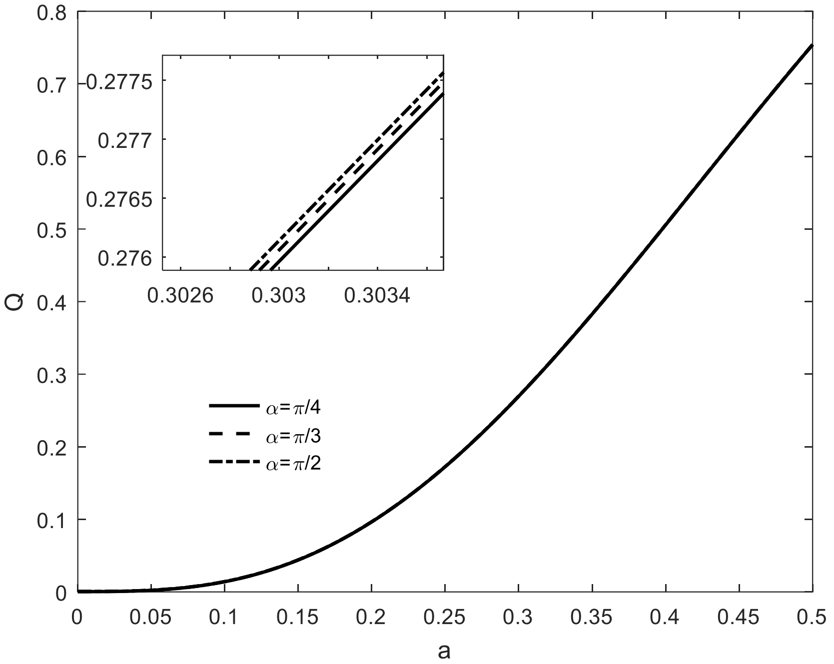

Figure 3 . Variation of flux Q with radius a for different values of the inclination parameter α **.**

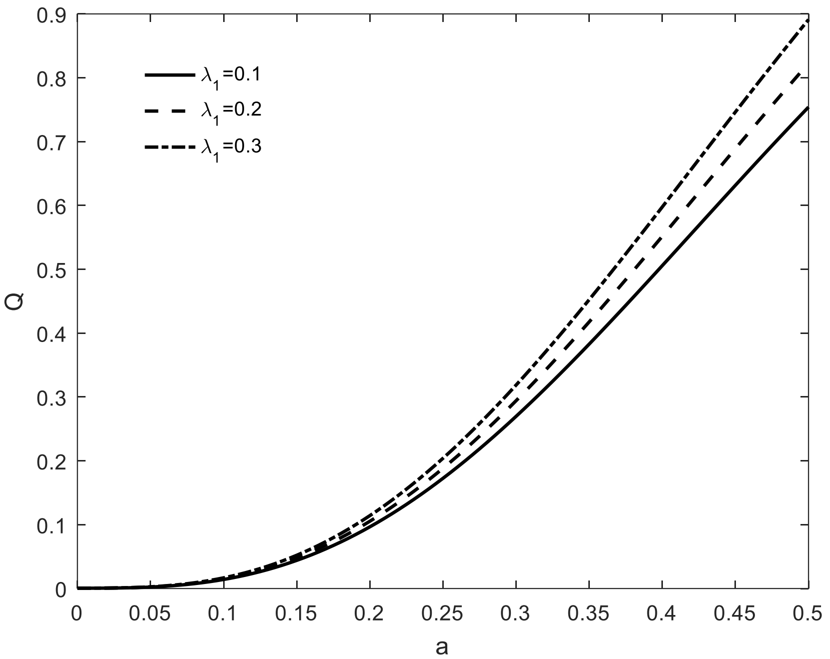

Figure 4. Variation of flux Q with radius a for different values of the Jeffrey parameter

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

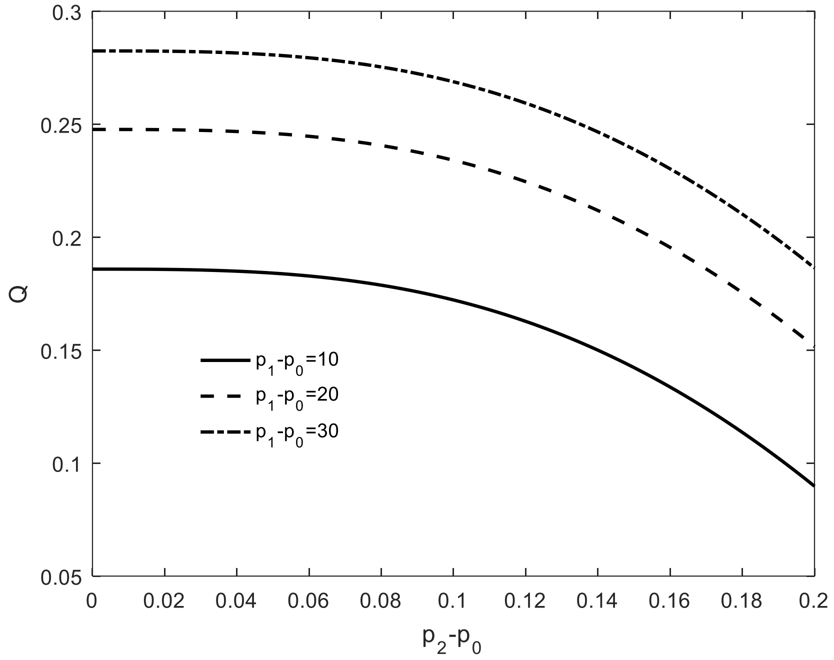

Figure 5. Variation of flux Q with $p_2 - p_0$ ***for different values of*** $p_1 - p_0$.

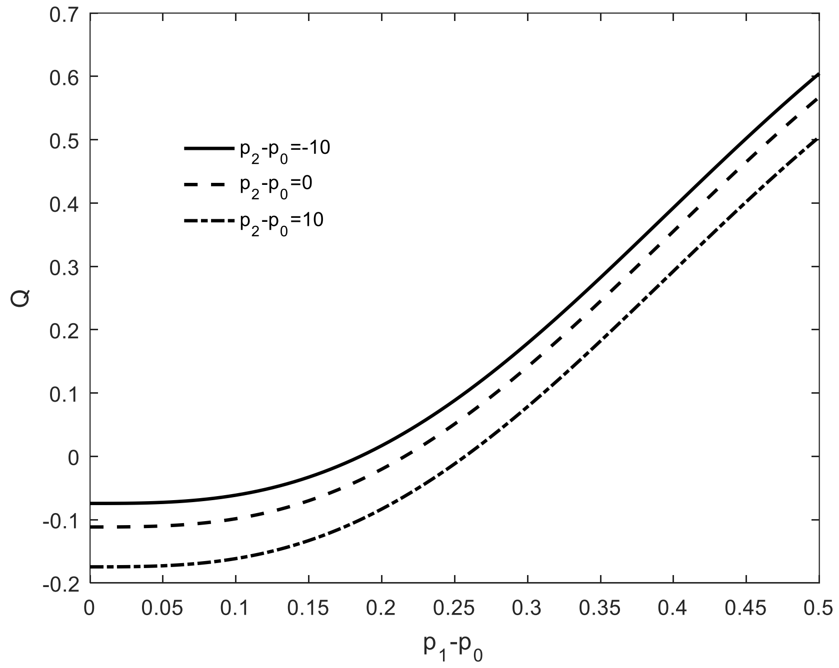

Figure 6 Variation of flux Q with $p_1 - p_0$ ***for different values of*** $p_2 - p_0$.

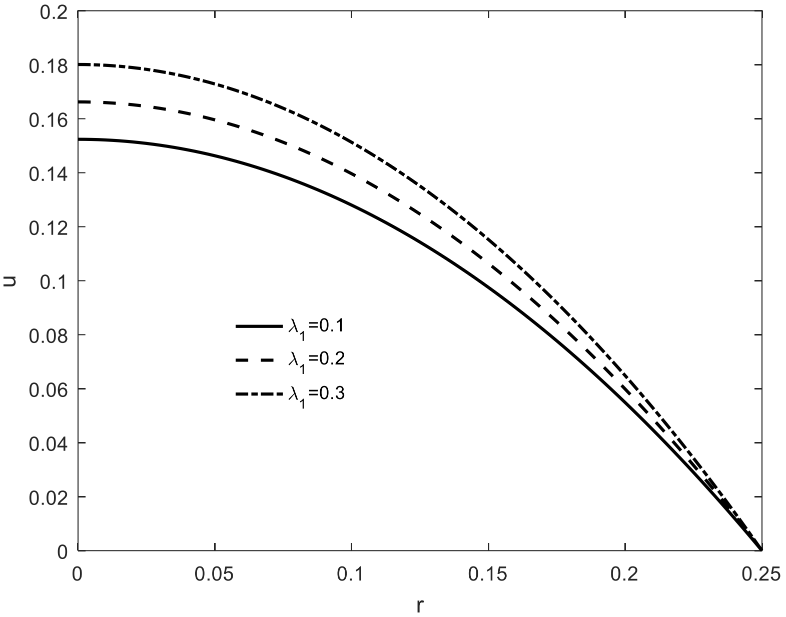

Figure 7. Velocity profile for different values of Jeffrey parameter λ_1 ***with inlet pressure*** $p_1 - p_0 = 10, a = 0.25$

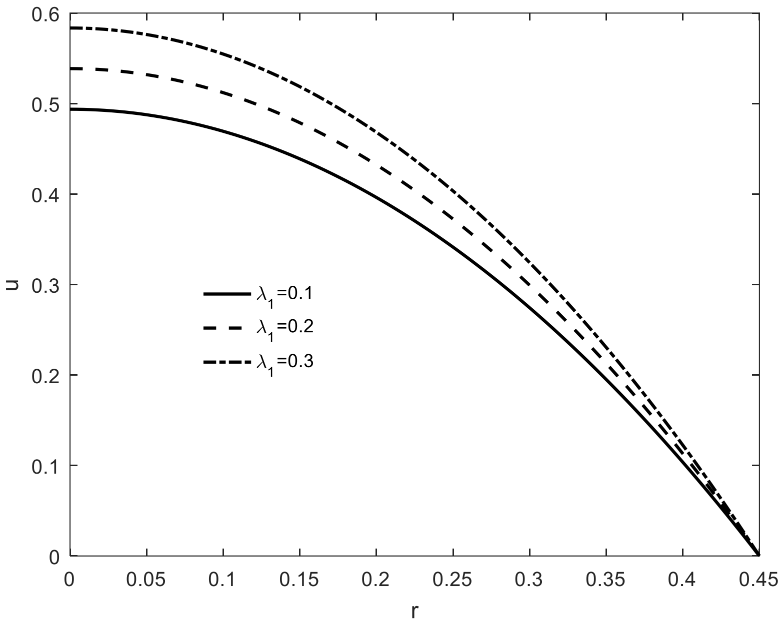

Figure 8. Velocity profile for different values of Jeffrey Parameter λ_1 ***with outlet pressure*** $p_2 - p_0$ ***=1000, a=0.45.***

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

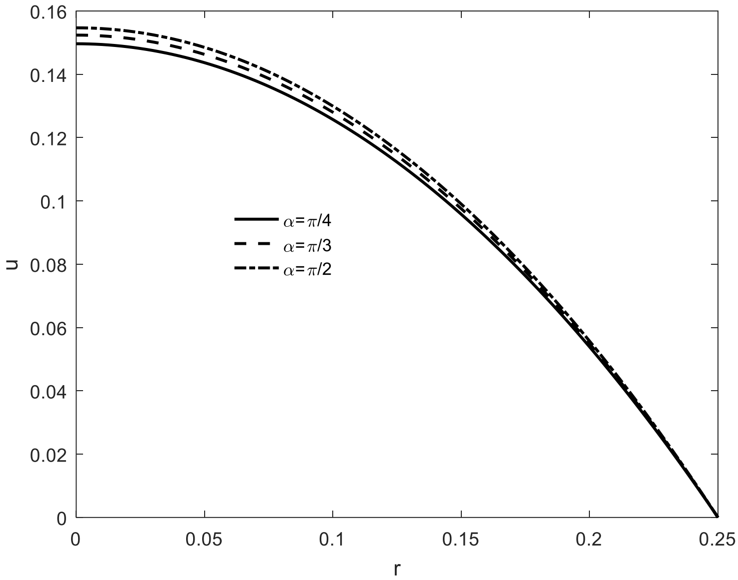

Figure 9. Velocity profile for different values of inclination parameter α ***with inlet pressure*** $p_1 - p_0 = 10, a = 0.25$

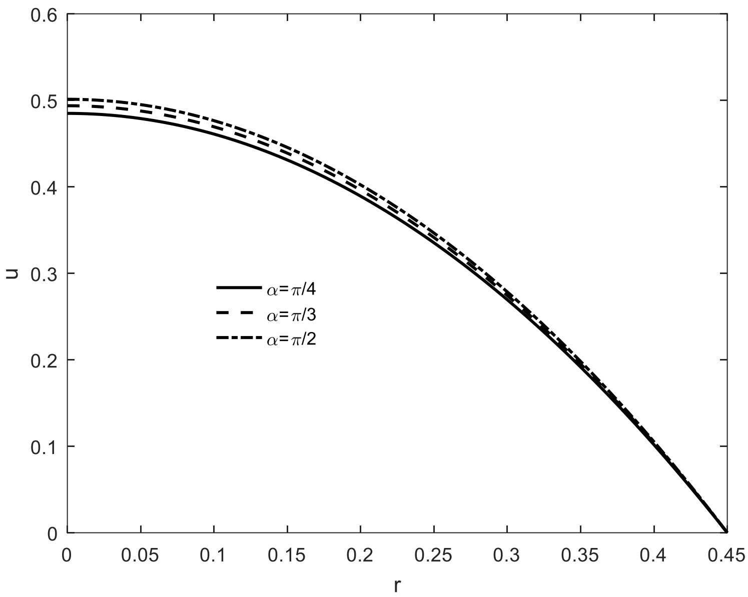

Figure 10. Velocity profile for different values of inclination parameter α ***with outlet pressure*** $p_2 - p_0$ ***=1000, a=0.45.***

VII. CONCLUSIONS

In this paper, we modeled the effects of elasticity on the Poiseuille flow of steady laminar incompressible Jeffrey fluid in an inclined elastic tube. The variation of flux and velocity field are analyzed with respect to various physical parameters, namely inclination parameter, Jeffrey parameter, elastic parameters, inlet and outlet pressures. We find some interesting facts:

1. The flux increases when there is an increase in elastic parameters.
2. The flux increases with the increase in inclination parameter.
3. The flux increases with the increasing values of Jeffrey parameter.
4. The inlet and outlet pressures have opposite behavior on the flux and
5. The velocity increases with the increasing values of Jeffrey and inclination parameters respectively

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

VIII. REFERENCES

[1] T.Young, Hydraulic investigations, "subservient to an intended croonian lecture on the motion of the blood" in Philos. T. R. Soc. Lond., 9 Vol. 8, 1808, pp.164-186.
[2] S.I.Rubinow, J. B.Keller, "Flow of a Viscous Fluid Through an Elastic Tube with Applications to Blood Flow" in J. Theor. Biol. Vol. 35, 1972, pp.299-313.
[3] J. Lahav, N. Eliezer and A.Silberberg, " Gel-walled cylindrical channels as models for the microcirculation: Dynamics of flow" in Biorheology, Vol. 10, 1973, pp.595-604.
[4] P. Krindel and A .Silberberg, "Flow through gel-walled tubes" in J. Colloid Int. Sci. Vol. 71, 1979, pp.39-50.
[5] V .Kumaran, "Experimental studies on the flow through soft tubes and channels" in Sadhana , Part 3, Indian Academy of Sciences, Vol. 40, 2015, pp.911-923.
[6] Mark Peter Rast, "Simultaneous solution of the Navier-Stokes and elastic membrane equations by a finite element method" in International Journal for Numerical methods in fluids, Vol. 19, 1994, pp.1115- 1135.
[7] J. R.Womersley, "Oscillatory flow in arteries: the constrained elastic tube as a model of arterial flow and pulse transmission" in Physics in Medicine and Biology, Vol. 2, No. 2, 1957.
[8] H.A.Erbay, H.Demiray, "Finite axisymmetric deformation of elastic tubes: an approximate Method" in Journal of Engineering Mathematics, Vol. 29, 1995, pp.451-472.
[9] Dalin Tang, David Anderson, Sophie Biz, N. Ku. David, "Study viscous flow in constrained elastic tubes subjected to a uniform pressure" in Int. J. Numer.Meth.Engng. Vol. 41, 1998, pp.1391- 1415.
[10] Vajravelu Kuppalapalle, Sreenadh Sreedharamalle, Devaki Palluru, Prasad Kerehalli V, "Mathematical model for a Herschel-Bulkley fluid flow in an elastic tube" in Cent. Eur. J. Phys. Vol. 9, No. 5, 2011, pp.1357-1365.
[11] Vajravelu K, Sreenadh S, Ramesh-Babu V, "Peristaltic pumping of a Herschel-Bulkley fluid in an inclined tube" in Appl. Math. Comput. Vol. 169, 2005, pp.726.
[12] Taha Sochi, "The flow of Newtonian and Power law fluids in elastic tubes" International journal of non-linear mechanics, May 2014 .
[13] S. Sreenadh, A.Raga Pallavi, Bh.Satyanarayana, "Flow of a Casson fluid through an inclined tube of non-uniform cross section with multiple stenosis" in Advances in Apllied Science Research, Vol. 2, No. 5, 2011, pp.340-349.
[14] Y.V.K.Ravikumar, S.V.H.N. Krishna Kumari, M.V.Ramana Murthy, S.Sreenadh, "Unsteady peristaltic pumping in a finite length tube with permeable wall" in Trans. ASME Jr of Fluids Engg., Vol. 132, 2010, pp.101-110.
[15] T. Hayat, N. Ali, S. Asghar, A. M. Siddiqui, "Exact Peristaltic Flow in Tubes with an Endoscope" in Appl. Math. Comput. Vol. 182, 2006, pp.359-368.
[16] S. Sreenadh, P. Devaki, Diwakar reddy , G. Krishnaiah, "Unsteady flow of a Jeffery fluid in an elastic tube with stenosis" in FDTT, IPCSIT, Vol. 33, 2012, pp.136-142.
[17] K. Vajravelu, S. Sreenadh, P. Lakshminarayana, "The Influence of Heat Transfer on Peristaltic Transport of a Jeffrey Fluid in a Vertical Porous Stratum" in Commun. Nonlinear Sci. Numer. Simul. Vol. 16, 2011, pp.3107–3125.
[18] K. L. Jyothi, P. Devaki, S. Sreenadh, "Pulsatile Flow of a Jeffrey Fluid in a Circular Tube having internal Porous Lining" in Int. J. Math. Arch. Vol. 4, 2013, pp.75-82.
[19] M. Kothandapani, S. Srinivas, "Peristaltic Transport of a Jeffrey Fluid under the Effect of Magnetic field in an Asymmetric Channel" in Int. J. Non-linear Mech. Vol. 43, 2008, pp.915-924.
[20] S. K. Pandey, D. Tripathi, T. Fang, "Unsteady Model of Transportation of Jeffrey-fluid by Peristalsis" in Int. J. of Biomath.Vol. 3, 2010, pp.473-491.
[21] A. Kavitha, R.Hemadri Reddy, R.Saravana, S. Sreenadh, "Transport of a Jeffery fluid in contact with a Newtonian fluid in an inclined channel" in Ain Shams Engineering Journal, Vol. 8, No. 4,Dec 2017, pp.683-687.
[22] S.Sreenadh, B. Sumalatha, A.N.S. Srinivas, "Effect of elasticity on Hagen-Poiseuille flow of a Jeffrey fluid in a tube" in International journal of Mechanical Engineering and Technology, Vol. 8, No. 8, 2017, pp.1343-1353

CITE AN ARTICLE

Sumalatha, B., & Sreenadh, S. (n.d.). POISEUILLE FLOW OF A JEFFREY FLUID IN AN INCLINED ELASTIC TUBE. *INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY*, *7*(1), 150-160.